

## **Creative Problem Solving in Math**

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### **INTRODUCTION**

One Friday, a professor gave a take home test to his students. It consisted of three math problems. He explained that the first two problems were very difficult, but the third had never been solved and it was for extra credit. To get credit for the third problem, a student would not have to get a solution, but show evidence of good thought processes.

Shortly thereafter, one of his students entered the class. Since he was late, he copied the assignment off the board and received no further instruction or explanation.

This student, whose grades were marginal, worked diligently all weekend, in an attempt to raise his average. With much effort he solved the first two problems and with extreme effort he solved the third. He brought his assignment to his teacher that Monday and told him that he must need tutoring because solving all three problems took him all weekend. The amazed teacher explained the enormity of the student's accomplishment to him as well as to the whole class.

My college psychology professor used this anecdote to illustrate the effect of mindset, expectation, and attitude, on creative problem solving. Not knowing that he wasn't expected to solve the problem freed the student from any self-perceived limitations and allowed him to apply maximum effort.

As a math resource and math inclusion teacher, I work with students that have that same constricted mind-set. They already believe that the answer can't be found unless it jumps out at them. They allow their own low expectations and perceived limitations to prevent them from reaching their potential.

In basic calculations, the solution steps are predictable. In this area they usually function well. However, when the student takes algebra, and especially geometry or fundamentals of math (special education), where thinking/problem solving skills are required, they have great difficulty.

The curriculum unit topic I have chosen is Creative Problem Solving Skills (CPS) in Math. I believe that focusing on this topic and teaching it as a skill, followed by reinforcing this skill throughout the different units during the semester, can be an invaluable asset to the students' functioning in school. There are many projects that involve creative solutions such as research papers, science and social studies fairs,

creative writing and others. As well, when confronted with an essay question, the student can employ CPS to select and develop his response. Unlike rote memorization, CPS expands rather than constricts the thought processes.

## **OBJECTIVES**

1. Students will be able to state the sequence of steps involved in Creative Problem Solving.
2. Students will be able to brainstorm (generate possible solutions to a problem).
3. Students will be able to evaluate the merits of possible solutions in terms of their adherence to criteria.
4. Based on a set of criteria, students will be able to make an optimal choice.
5. Students will be able to develop a plan to implement the solution.
6. Students will be able to articulate how Creative Problem Solving can be used in various areas.

## **STRATEGIES**

In his classic book, Applied Imagination, (1963), Osborn states that the creative problem solving process has three procedures: 1) Fact-Finding, 2) Idea Finding and 3) Solution Finding. He explains that the fact-finding portion involves defining the problem and "...gathering and analyzing pertinent data." Idea finding concerns generating and modifying ideas. Lastly, solution finding is the evaluative process that culminates in finding a final solution (p. 86).

Roger Van Oech, in his book, A Whack on the Side of the Head (1990), sees the creative process as having "...two main phases...an imaginative phase and a practical one." In the imaginative phase, you generate and play with ideas. In the practical phase, you evaluate and execute them (p. 38).

More recently, Treffinger's study (1995), "Creative Problem solving: Overview and Educational Implications", reviews the later trends in creative problem solving. According to his article, CPS is ...

Representing process dimensions in a natural, rather than in a contrived way.

Undergoing a transformation from a prescriptive to a descriptive approach.

Becoming more flexible and responsive to task, contextual, personal, methodological and meta-cognitive consideration.

There is a departure from a "...process view that prescribes a fixed number or kind of particular steps or strategies that must be applied in a fixed sequence (pp.304-305).

In other words, the concept is more dynamic and interactive. Although this appears to be the more current trend, I have chosen to follow a more traditional model, because experience suggests that a more structured procedure works best at first. As the

students become more skilled, a more dynamic approach becomes possible because they have internalized the procedures.

I will be using an amalgam of Van Oech's and Osborn's procedures. This will be a four-step process. The stating of the problem (fact finding) will be uniform throughout the classroom groups. They will all be working on a specific problem. The four phases will be: 1) Clarification of the problem 2) Brainstorming 3) Evaluation and Selection and 4) Implementation.

Clarification of the problem involves making sure all students understand what the solution of the problem calls for. This will include a review of the criteria for successful completion of the project.

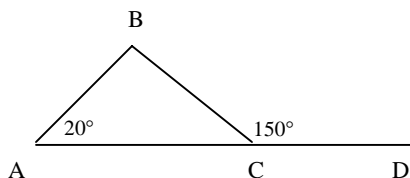
During Brainstorming, students will generate as many ideas as possible. Idea play will be encouraged with an emphasis on suspending judgement and criticism in order to encourage free flow and stimulate maximum output.

During the Evaluation/Selection phase the group members will evaluate the pros and cons of each suggestion, modifying where possible and eliminating when necessary, with the goal of the group's deciding on one selection.

The last phase, Implementation, will consist of developing a plan to implement their choice.

Although creative problem solving traditionally deals with problems that have multiple solutions, such as those found in management, math usually involves only one solution. But, Geometry and other math units often pose problems where there are multiple ways of coming to the same solution. Here is an example:  
If Angle BCD =  $150^\circ$  and Angle A =  $20^\circ$ , find Angle B.

**Figure 1**



In this problem, you subtract 150 from 180 giving you 30, since angle BCD and angle BCA comprise a straight line which equals 180 degrees. You then add  $30 + 20$  and subtract the answer (50) from 180 (the sum of the degrees of the angles in a triangle) to give you angle B which equals 130.

You could also identify angle BCD as an exterior angle and as such it would be equal to the sum of angle A and angle B. By subtracting 20 (angle A) from 150, you would get 130. These two different methods arrive at the same solution.

Thus, it has been my experience that the existence of more than one way to solve a problem, becomes a problem in itself for our students. Rather than freeing them and allowing them more creativity in their approach, it confounds them.

For example, in my resource math class I had a student who lacked basic skills and was unable to keep up. To help him, I put him on a remedial plan to address these deficiencies. Every week one of my other students asked me, "Why can't I do what *he's* doing?" rather than the day's lesson or, "Why does *he* get to do that?", as if working on something they already know is a reward.

By familiarizing my students with the steps in creative problem solving, my objective is to help them overcome this difficulty and enable them to apply this skill to math and other areas.

As a way of testing the feasibility of this curriculum unit, I did my own abbreviated pilot study using my resource fundamentals of math 1B class. The project was a poster about percents.

In order to clarify the problem the class generated a list of areas where percents are used (e.g. weather, sports, discounts, crime reports, etc.). I then reviewed the following criteria for successful completion.

1. The poster must have a title involving percents.
2. It must have at least one illustration (picture).
3. Two students must work together and participate in the project.
4. The poster must be completed by the end of the unit.
5. Each student must do a rough draft and get approval before doing it on a poster board.

Both lists were left on the board for future reference and they were given one week to complete the task.

My students were intrigued by the notion of a project, but predictably had difficulty in phase two (Brainstorming). I circulated among the groups and had to prompt them in order to generate ideas.

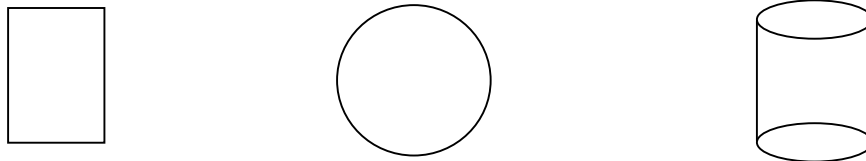
In phase three (Evaluation/Selection), their approach was not methodical. They appeared to decide upon their topic by trial and error (If they couldn't come up with a concept, they chose another idea.).

Phase four (Implementation) also gave them difficulty. It was hard for them to put their ideas on paper. However, most of them were able to complete the project successfully, based on my criteria.

From this experience, I decided that it was necessary to spend the first day of the unit on the process itself rather than having them initially experience it and then formulate what occurred afterwards.

On the first day I will introduce the C.P.S. unit by explaining what it is and how it could be useful to them in math and other areas. I'll stress the importance of valuing everyone's ideas. To get them started I will display three shapes found below and ask them to choose the one that is different (adapted from A Whack on the Side of the Head). (p. 22).

**Figure 2**



The idea is that it depends on how you look at it. Each figure could be the different one. The cylinder is different because it is the only three dimensional shape. The circle is different because it is the only shape that does not contain a rectangle (the side of a cylinder). The rectangle is different because it is the only shape that does not contain a circle. This exercise underscores the need for understanding that there is more than one way to look at things and thus valuing each person's perspective.

The list of steps will be placed on the board and an overview of the types of problems that CPS deals with will be reviewed, as will the group process. The concept of multiple solutions or approaches will be emphasized. Vocabulary will be explained (Criteria, phases, clarification, brainstorming, selection, and implementation, evaluation).

The students will then be divided into groups with three to five students per group, to give everyone a chance to participate. The teacher's role will be explained as that of a facilitator. The ideas must come from the group.

#### CLARIFICATION OF THE PROBLEM

The students will receive a copy of three projects with three different sets of criteria that will thoroughly be reviewed. They will then be given another sheet with solutions for the problem and each group will have to decide whether or not each solution met the criteria. This will give the students a better sense of the concept.

### BRAINSTORMING

The groups will be given (one at a time) a list of broad topics and they will have to generate as many responses as possible. It will be done as a game. The winner is the group that has the most responses. There will be an emphasis on quantity and non-judgmental attitude.

### EVALUATION/SELECTION

Using the lists that the group generated, they will be given a list of criteria and will eliminate and modify their choices until they are left with one choice.

### IMPLEMENTATION

Here the group will decide upon how they will choose to do the project. Since this is not the actual project, one representative from each group will describe how the group chose to implement it.

There will be emphasis on making each part of this unit as much like a game as possible.

On the second day, the class will again be divided into groups. The teacher can choose whether the groups are permanent or changing and whether the students or the teacher choose the group composition. They will have a specific project. An example would be to design a board game. Upon completing the project, the students will actually play their game to see how it works and make modifications as needed. This would demonstrate the reason for adherence to the rules. A list of criteria (rules) is extremely important. The number is significant. Too few rules would lead to an unclear project concept and would not give the student practice with adhering to the construct. Too many rules would be too difficult and restrictive to follow thus inhibiting creativity. Some sample rules for creating a board game would be:

- The game must have a title.
- The game must have a beginning and an end.
- There must be a way to get around the board and the board itself must be involved.

On the succeeding days the lessons will incorporate more and more math-specific skills, beginning with basic math and leading up to geometry. They could include two or three-dimensional solutions. At the end of the each class the last fifteen minutes will be reserved for debriefing. During this portion of the class each group will choose a spokesperson to explain how their group completed the project. Other groups can ask one or two questions.

If this skill is not practiced throughout the year it will be lost. It will be up to the teacher to look at each unit and decide how to include even a small creative problem-

solving lesson in each unit she teaches. As with anything, practice leads to improvement and enough practice leads to mastery.

These lessons are designed for block scheduling (one and a half-hour classes). Adjustments can be easily made for shorter time periods by eliminating repetition of any portion of the class or dividing the lesson into two or three segments. As well, the teacher can use his own judgement as to whether the class needs more practice anywhere he feels the students are not grasping the concept.

## **LESSONS**

### **Day 1**

#### ***Introduction***

Introduce the topic with an introduction such as this one adapted from Recall Enhancement Routine (Schumaker, Bulgren, Deshler and Lenz).

*“Today I will be showing you a method to help you think and solve problems more creatively. It is called Creative Problem Solving (CPS) and you will be using it at different times throughout the year. Once you learn this procedure you will find that it won’t just help you in your math classes but in other subjects as well. I’ll review the steps with you today and you’ll have an opportunity to practice each step so that you could become more familiar with it. We will be using it in groups so it is very important that within your group each person gets a chance to participate and that each person’s ideas are valued.” (p. 26)*

#### ***Warm Up***

Three figures are displayed on the overhead (see figure 2). The students are asked which figure is different from the others. As the students offer their ideas and explain their choices, the teacher responds positively while pointing out why their answer is incorrect, (unless a student figures out that there is no correct answer). The teacher makes the connection that all contributions in a group are important.

#### ***Vocabulary and Procedures***

The teacher explains the concept of CPS and defines all relevant terminology and steps. Criteria, clarification, brainstorming, evaluation, selection and implementation are examples of words the teacher should explain as well as any other she feels that are indicated depending on the class level. The teacher also gives examples of where CPS can be used and elicits additional examples from the class, making sure to include an explanation of how it can be used in school. At the end of this section, the teacher plays

a game with the students (2 teams). She gives a term that she reviewed and a team member must give an example or definition of that term.

For the remainder of the class the students will be divided into groups of three to five members. The teacher explains her role as that of facilitator. They work on the following problems:

#### CLARIFICATION OF THE PROBLEM

The students will receive three projects with three different sets of criteria, all of which will be thoroughly reviewed. These can be adjusted to things familiar to the students in your particular class.

Example-"Write a one-minute speech persuading the school board to change one thing in your school. Criteria- it must be realistic; it cannot be destructive or negative and it must be specific."

In this segment the teacher is not brainstorming with the students but giving correct and incorrect examples of ideas to see if they understand adherence to criteria.

#### BRAINSTORMING

The students are learning how to generate as many ideas as possible. The teacher must stress the non-critical attitude of the group and explain that often ideas that appear to have nothing to do with the criteria are useful in generating other solutions, even if they are not appropriate as the solution themselves.

In order to illustrate the importance of brainstorming and how seemingly unrelated ideas can lead to a solution, the teacher can use this example, or something from her own experience.

In a marketing class, I participated in a group. Our project was to develop our own product and marketing plan. Since brainstorming was not the time to criticize, our group agreed to allow a free flow of ideas, no matter how ridiculous, or "off track" they seemed. We thought about products we felt there was a need for but had never been manufactured, or any inventions we dreamed up as children. We then discussed other peoples' inventions and soon began reminiscing about the days before the microwave oven. One of our group members said offhandedly, "Too bad they don't have something like that to chill foods in a hurry." We all looked at each other and knew we found our idea. We named it "Microchill" and it was easily approved by our professor. Had we not allowed the free flow of ideas or told each other not to talk about things that already exist, our group never would have come up with that idea.

Example-"Without putting any limits on time or money, think of as many things as you can that would improve the quality of life in Houston. Each response will be worth a point and the team that gets the most points wins a prize."

### EVALUTION/SELECTION

Using the example from the brainstorming exercise or a new list generated by the teacher with a corresponding list of criteria, the students will learn how to be critical and come to a consensus decision on one optimal choice.

Example-using the above list the teacher puts criteria on the board, such as “Your solution must be free or a minimal cost and must be able to be implemented within one month.” The teacher then states, *“Now your task has changed. You must be more selective. Look at each selection and eliminate all those that do not fit the criteria. This phase is not brainstorming. You are evaluating each selection to see how it fits with the criteria. When you are finished, you should have only one choice.”*

### IMPLEMENTATION

During this phase the students will construct a rough plan to implement their final choice from the previous phase. *“You are now at the final phase or procedure, called Implementation. You are deciding how to do your project, what form it will take and what it will actually look like. It can be done in a paragraph, a picture or a diagram (graphic organizer) describing what you intend to do. Each group must select a spokesperson. His or her job is to explain the solution to the rest of the class. During this time the other groups must pay careful attention because the rest of you will be giving feedback and constructive criticism, or you could ask questions about anything that doesn't make sense to you.”*

Depending upon the teacher’s own creativity, and the particular students she has, she can adapt the number and types of exercises used. At the end of this first lesson the teacher reviews the steps in CPS and the terminology, using the class responses to the exercises as examples and prompting the students to give their own examples.

### **Day 2**

The teacher either allows the students to form their own groups of three to five students or chooses the groups, himself. He then introduces the lesson by reviewing the steps in Creative Problem Solving and moves into the actual project. This lesson will serve as a review of the basic concepts and will allow the students actual practice with CPS.

### CLARIFICATION OF THE PROBLEM

*“Today’s project is a poster on the creative problem solving process. The finished products will be displayed around the room as reference material. Criteria for the project are as follows: 1) The poster must have a title involving Creative Problem Solving. 2) It must have at least one illustration. 3) It can depict one or more phases of the process or the whole process. 4) It must have colors. 5) It must be original. 6) A rough draft must be submitted for approval prior to doing it on a poster board. Before you progress through the phases we*

*have discussed, you must get approval. I'll be coming around the room to assist you, but the ideas must come from you."*

### BRAINSTORMING

Each group develops a list of ideas about how to do a poster on Creative Problem Solving.

*"Now that everyone is clear on the project, each group will brainstorm possible solutions. Remember, during this phase you should not be criticizing each other's ideas. You never know when something that sounds crazy will connect to a solid idea. Have fun with it. If your group gets stuck, think wilder and crazier."*

### EVALUATION/SELECTION

The group reviews each suggestion (in this project there may only be two or three) keeping in mind the criteria, and selects the best choice.

*"Now that each group has generated a list, the time has come for you to be more selective. Keeping the criteria in mind, review each suggestion and see which is the best fit."*

### IMPLEMENTATION

The group specifies the layout of the illustrations, captions, titles, colors etc. culminating in a rough draft for teacher approval.

*"Now that you have narrowed down your selection to one choice, you must decide how best to complete your project. When the group has agreed on how you will do it, submit the proposal to me for approval. Remember, there is not one right way. You can use markers, paints, pictures from magazines or anything else that can be attached to a poster. You can also be creative in the type of writing you use (e.g., printing, script, stencils, calligraphy etc.)"*

As the groups become proficient in CPS they become more independent, and approval can be eliminated. The teacher's role is to circulate among the groups and assist by prompting, clarifying and using questions to move the group forward.

## **Day 3**

The teacher introduces the lesson with the following: *"You should now be more knowledgeable about the steps in Creative Problem Solving. Today we will combine this technique with something you should all be familiar with, board games. Lets name some and see if we can find any similarities among them. "*

### CLARIFICATION OF THE PROBLEM

The teacher leads the class through a list of attributes which ultimately become the criteria for creating the game: 1) It must have a title. 2) It must have a beginning and an end. 3) It must have simple directions or the group must be able to easily explain how to play. 4) It must have a theme. 5) It must have pieces that

move around the board. 6).The way to get around the board must be specified (dice, spinner etc.). 7) There must be bonuses or penalties specified on the board and in the directions (e.g. certain color squares indicate an extra turn or losing a turn, landing on a square occupied by your opponent puts a piece back to the beginning, etc. 8) Each group must submit a rough draft prior to actually creating the game.

Depending on the level of the class, the teacher can use more or less challenging criteria. Also, it easily can be expanded into a two period lesson. The teacher should leave at least fifteen minutes for the groups to actually play their games and make any necessary modifications.

If the curriculum being taught involves probabilities, the teacher may choose to save this lesson for that unit, where the students can figure out probabilities of landing on certain squares and how those probabilities differ with one or two dice, or a spinner.

### BRAINSTORMING

Students come up with different ideas for a game. They could begin with games they already know, board designs they are familiar with or mythical places. This project easily lends itself to CPS.

*"Yesterday's project was a poster. Today's project is a little more complicated, but since you have all played or seen board games, you already have the concept in your mind. You may want to begin with games you already know and branch out from there. You can use a favorite story or movie. You can create your own place or use a place that exists. You can even think about a favorite subject or something that you love to do outside of school. You are looking for a theme-something that unifies the game. Remember, keep your ideas flowing."*

### EVALUATION/SELECTION

During this phase they look at the criteria to make a final choice.

*"It is now time to look at the list of criteria for the game. It may help to eliminate those ideas which are obviously a poor fit. When you are down to two or three ideas, look at each criterion and see which idea can be used with the fewest modifications."*

### IMPLEMENTATION

The group makes a rough sketch of the game board and is either able to explain how the game is played or writes down simple directions.

*"You are now ready to design the game itself. Remember to review the criteria and that all parts of the game must relate to the theme. When your group has completed a rough draft, submit it to me for approval. During the last part of this class period each group will play its game. Make any necessary changes, as time permits."*

## Day 4

In this lesson, the teacher will apply Creative Problem Solving to a math problem. Depending upon when CPS is introduced, the teacher can adjust the example to fit the unit he is teaching. For example, if CPS is taught at the very beginning of the school year, the teacher may prefer to use an example from basic math. He introduces the lesson with the following script:

*“As you know, this week we have been studying Creative Problem Solving. We began by discussing vocabulary and reviewing the four phases namely, Clarification of the problem, Brainstorming, Evaluation/Selection and Implementation. We have used CPS to create a poster and a board game. Today you are ready to apply it to solving a math problem.”*

Show the class figure 1 found in this paper.

### CLARIFICATION OF THE PROBLEM

*“To solve this problem, you must use information you already know about triangles and angles. You will need to do more than one step. It is also important to keep in mind that there is more than one way to solve this problem, but only one solution. Remember you are looking for the number of degrees in angle B.”* There will be those of you who immediately see the answer. If you see it, resist the urge to solve and go along with the four phases. Remember, you are learning a technique which can be used with more complex problems.

The teacher should encourage the brighter students in each group to act as leaders, guiding the group through the four phases

### BRAINSTORMING

*“Without doing any math calculations, brainstorm everything you know about angles and triangles.”*

Here the teacher monitors the groups' responses and prompts them when necessary to assure that all relevant information is elicited. This approach is especially beneficial for students who draw a blank when confronted with this type of problem.

### EVALUATION/SELECTION

In this stage the students eliminate unnecessary and unhelpful information and begin to see possible solutions.

*“It is now time to solve the problem. Look at all the information you listed and see if you can eliminate anything that would not help you. For example, if you listed various types of triangles classified by their sides such as equilateral, scalene and isosceles, this would not be helpful and could be crossed off your list. If you to see some ideas for solving the problem make note of them. Remember, this is not the final phase.”*

### IMPLEMENTATION

During this phase the groups are actually doing the math calculations. They use the information generated during the previous phase as well as any solution strategies.

*“Now it is time to actually solve the problem. Use the information you listed during the last two phases. Remember there is more than one step. If you are still having trouble solving the problem, return to your Brainstorming and Selection/Evaluation lists.”*

At the end of this activity the teacher allows a spokesperson from each group to share their experiences through the phases. As well, the class discusses how CPS can be used in other units and subjects.

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